Indices

1) i) $\lambda \in Sp(A) \iff A - \lambda I$ non inversible

$\iff A - \lambda I$ non injective

ii) $A = P.B.P^{-1} \Rightarrow A - \lambda I = P(B - \lambda I).P^{-1}$

iii) Théorème du rang.

2) a) $P_A(\lambda) = -\lambda(\lambda-1)(\lambda-2)$

b) $\{0, 1, 2\} = Sp(A)$

c) $\longrightarrow E_0 = \mathrm{Vect}\begin{pmatrix}1\\0\\1\end{pmatrix}$; $E_1 = \mathrm{Vect}\begin{pmatrix}1\\1\\1\end{pmatrix}$; $E_2 = \mathrm{Vect}\begin{pmatrix}-1\\1\\0\end{pmatrix}$

d) $\det_{B_C}(S) \neq 0$; $S = \left(\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\0\end{pmatrix}\right)$

e) $P^* = \begin{pmatrix}1 & 1 & -1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{pmatrix}$; $P^{-1} = \begin{pmatrix}-1 & -1 & 2\\ 1 & 1 & -1\\ -1 & 0 & 1\end{pmatrix}$

$\cancel{(f)\ \mathrm{mat}_{B_C}(f) = P.\mathrm{mat}(f)}$

f) $A = P.D.P^{-1}$; avec $D = \begin{pmatrix}0 & & 0\\ & 1 & \\ 0 & & 2\end{pmatrix}$

h) $A^n = P.D^n.P^{-1}$.

$\forall n \geq 1$, $A^n = P.\begin{pmatrix}0 & & 0\\ & 1 & \\ 0 & & 2^n\end{pmatrix}.P^{-1} = \ldots$

i) Posons $X_n = \begin{pmatrix}x_n\\y_n\\z_n\end{pmatrix}$; on a $X_0 = \begin{pmatrix}1\\0\\-1\end{pmatrix}$.

$(\forall n \in \mathbb{N}, X_{n+1} = AX_n) \Rightarrow \forall n \in \mathbb{N}, X_n = A^n X_0$.

$\Rightarrow \begin{pmatrix}x_n\\y_n\\z_n\end{pmatrix} = \left(\text{voir h)}\right)\begin{pmatrix}1\\0\\-1\end{pmatrix} = \ldots$